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PI and PID Controller Tuning for Integral plus Time Delay Plants

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Abstract: *This paper describes and compares PI and PID analog controllers tuning methods for integration processes with time delay. The integrating plants are frequently encountered in a process industry and their control is often problematic. The uses of the described tuning methods are shown in the example.*

Keywords: *controller tuning, time delay, PID, IPTD*

1 Introduction

Integrating plants are very often encountered in industry. If the disturbances are in the plant output, then controller tuning is simple. If the disturbances are in its input (Fig. 1), then the controller with an integrating component must by used and controller tuning is rather difficult. In this paper the integral plus time delay (IPTD) plant is considered with the transfer function

$$G_P(s) = \frac{k_1}{s} e^{-T_d s}, \tag{1}$$

where k_1 is the plant gain, T_d – the time delay.

Mathematical model of the IPTD plant is very simply, it contains only two parameters k_1 and T_d, which can be determined on the basis of the plant step response.

By reason of structural instability a I controller cannot be used. Therefore PI and PID analog controllers are only considered further.

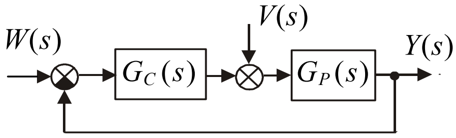

Fig. 1 – Control system scheme

In Fig. 1 the symbols W, V and Y are the transforms of the desired, disturbance and controlled variables, G_C and G_P – the controller and plant transfer functions.

2 Controller tuning methods

ZN method (Ziegler – Nichols)

The ultimate (critical) parameters Ziegler-Nichols method demands the ultimate gain k_{Pc} and ultimate period T_c determination. Both these ultimate parameters can be easily determined on the basis of the Nyquist stability criterion. For a proportional controller with the gain k_P the open-loop transfer function has the form

$$G_o(\mathrm{j}\omega) = \frac{k_1 k_P}{\mathrm{j}\omega} \mathrm{e}^{-T_d \mathrm{j}\omega} = \frac{k_1 k_P}{\omega} \mathrm{e}^{-\mathrm{j}(T_d \omega + \frac{\pi}{2})}. \tag{2}$$

On the basis of boundary stability conditions

$$\operatorname{mod} G_o(\mathrm{j}\omega_c) = 1, \quad \arg G_o(\mathrm{j}\omega_c) = -\pi \tag{3}$$

the values of ultimate parameters

$$\left.\begin{aligned} \frac{k_1 k_{Pc}}{\omega_c} &= 1 \\ -\left(T_d \omega_c + \frac{\pi}{2}\right) &= -\pi \end{aligned}\right\} \Rightarrow \begin{aligned} & k_{Pc} = \frac{\pi}{2 k_1 T_d} \\ & \omega_c = \frac{\pi}{2 T_d} \Rightarrow T_c = \frac{2\pi}{\omega_c} = 4 T_d \end{aligned} \tag{4}$$

can be obtained.

Now according to the ultimate parameters Ziegler-Nichols method [O´Dwyer, 2003] can be obtained:

PI controller

$$k_P^* = \frac{1}{2,2} k_{Pc} = \frac{\pi}{2,2 \cdot 2 k_1 T_d} \doteq 0,714 \frac{1}{k_1 T_d}$$

$$T_I^* = \frac{1}{1,2} T_c = \frac{4 T_d}{1,2} \doteq 3,333 T_d$$

PID controller

$$k_P^* = \frac{1}{1,7} k_{Pc} = \frac{\pi}{1,7 \cdot 2 k_1 T_d} \doteq 0,924 \frac{1}{k_1 T_d}$$

$$T_I^* = \frac{1}{2} T_c = \frac{4 T_d}{2} = 2 T_d$$

$$T_D^* = \frac{1}{8} T_c = \frac{4 T_d}{8} = 0,5 T_d$$

TL method (Tyreus – Luyben)

This method uses the ultimate parameters (4) too and therefore according to [O´Dwyer, 2003] there can be obtained:

PI controller

$$k_P^* = \frac{1}{3,2} k_{Pc} = \frac{\pi}{3,2 \cdot 2 k_1 T_d} \doteq 0,491 \frac{1}{k_1 T_d}$$

$$T_I^* = 2,2 T_c = 2,2 \cdot 4 T_d = 8,8 T_d$$

PID controller

$$k_P^* = \frac{1}{2{,}2} k_{Pc} = \frac{\pi}{2{,}2 \cdot 2k_1 T_d} \doteq 0{,}714 \frac{1}{k_1 T_d}$$

$$T_I^* = 2{,}2 T_c = 2{,}2 \cdot 4 T_d = 8{,}8 T_d$$

$$T_D^* = \frac{1}{6{,}3} T_c = \frac{4T_d}{6{,}3} \doteq 0{,}635 T_d$$

ChS method (Chidambaram – Sree)

The authors in their article [Chidambaram & Sree, 2003] derive the open-loop transfer function (see Fig. 1)

$$G_{wy}(s) = \frac{G_C(s) G_P(s)}{1 + G_C(s) G_P(s)} = \frac{k_1 (r_1 s^2 + r_0 s + r_{-1})}{s^2 + k_1 (r_1 s^2 + r_0 s + r_{-1}) \, \mathrm{e}^{-T_d s}} \mathrm{e}^{-T_d s}, \tag{5}$$

where the PID controller transfer function is considered in the form

$$G_C(s) = k_P \left(1 + \frac{1}{T_I s} + T_D s \right) = r_0 + \frac{r_{-1}}{s} + r_1 s, \tag{6}$$

$$k_P = r_0, \quad T_I = \frac{r_0}{r_{-1}}, \quad T_D = \frac{r_1}{r_0}. \tag{7}$$

For $r_1 = 0$ a PI controller can be obtained.

Time delay in the denominator (5) is approximated by the Padé expansion of the first order

$$\mathrm{e}^{-T_d s} \approx \frac{2 - T_d s}{2 + T_d s}, \tag{8}$$

i.e.

$$\begin{aligned} G_{wy}(s) &= \frac{k_1 (r_1 s^2 + r_0 s + r_{-1})(2 + T_d s)}{s^2 (2 + T_d s) + k_1 (r_1 s^2 + r_0 s + r_{-1})(2 - T_d s)} \mathrm{e}^{-T_d s} = \\ &= \frac{k_1 T_d r_1 s^3 + k_1 (2 r_1 + T_d r_0) s^2 + k_1 (2 r_0 + T_d r_{-1}) s + 2 k_1 r_{-1}}{T_d (1 - k_1 r_1) s^3 + (2 + 2 k_1 r_1 - k_1 T_d r_0) s^2 + k_1 (2 r_0 - T_d r_{-1}) s + 2 k_1 r_{-1}} \mathrm{e}^{-T_d s} \end{aligned} \tag{9}$$

By simple comparison of the numerator and the denominator terms, which stand in the same powers of the complex variable s, i.e.

$$\left. \begin{aligned} 2 r_0 + T_d r_{-1} &= 2 r_0 - T_d r_{-1} \\ k_1 (2 r_1 + T_d r_0) &= 2 + 2 k_1 r_1 - k_1 T_d r_0) \\ k_1 T_d r_1 &= T_d (1 - k_1 r_1) \end{aligned} \right\} \tag{10}$$

the PD controller parameter values can be obtained

$$r_0^* = \frac{1}{k_1 T_d}, \quad r_1^* = \frac{1}{2 k_1}, \quad r_{-1}^* = 0 \tag{11}$$

or

$$k_P^* = r_0^* = \frac{1}{k_1 T_d}, \quad T_D^* = \frac{r_1^*}{r_0^*} = \frac{T_d}{2} \tag{12}$$

The PD controller does not ensure a zero steady-state error for the disturbance step v, in cases of the plant input (see Fig. 1). Therefore, the authors, Chidambaram and Sree, propose the comparison of the numerator and the denominator terms by the "tuning" parameter α, i.e.

$$\left.\begin{aligned} 2r_0 + T_d r_{-1} &= \alpha(2r_0 - T_d r_{-1}) \\ k_1(2r_1 + T_d r_0) &= \alpha(2 + 2k_1 r_1 - k_1 T_d r_0) \\ k_1 T_d r_1 &= \alpha T_d (1 - k_1 r_1) \end{aligned}\right\} \tag{13}$$

On the basis of the equations (13) there can be obtained:

PI controller ($r_1 = 0$)

$$k_P^* = r_0^* = \frac{2\alpha}{k_1 T_d (1+\alpha)}$$

$$T_I^* = \frac{r_0^*}{r_{-1}^*} = \frac{T_d}{2}\frac{\alpha+1}{\alpha-1}$$

PID controller

$$k_P^* = r_0^* = \frac{4\alpha^2}{k_1 T_d (1+\alpha)^2}$$

$$T_I^* = \frac{r_0^*}{r_{-1}^*} = \frac{T_d}{2}\frac{\alpha+1}{\alpha-1}$$

$$T_D^* = \frac{r_1^*}{r_0^*} = \frac{T_d}{4}\frac{\alpha+1}{\alpha}$$

For the recommended tuning parameter value $\alpha = 1.25$ there can be obtained [Chidambaram & Sree, 2003]:

PI controller

$$k_P^* \doteq \frac{1{,}111}{k_1 T_d}$$

$$T_I^* = 4{,}5 T_d$$

PID controller

$$k_P^* \doteq \frac{1{,}235}{k_1 T_d}$$

$$T_I^* = 4{,}5 T_d$$

$$T_D^* = 0{,}45 T_d$$

MDP method (Multiple Dominant Pole)

The multiple dominant pole method supposes the existence of the stable real dominant pole with multiplicity increased by 1 more than the number of the adjustable controller parameters [Šulc & Vítečková, 2004].

For the characteristic quasipolynomial

$$N(s) = s^2 \mathrm{e}^{T_d s} + k_1(r_1 s^2 + r_0 s + r_{-1}), \tag{14}$$

of the closed-loop control system with the transfer function (5) the multiple dominant pole and the PID controller adjustable parameters can be obtained by working out the system of equations

$$\frac{\mathrm{d}^i N(s)}{\mathrm{d}s^i} = 0 \quad \text{for} \quad i = 0,1,2,3 \tag{15}$$

i.e.

$$s^2 \mathrm{e}^{T_d s} + k_1(r_1 s^2 + r_0 s + r_{-1}) = 0$$
$$T_d s^2 \mathrm{e}^{T_d s} + 2s\,\mathrm{e}^{T_d s} + 2k_1 r_1 s + k_1 r_0 = 0$$
$$T_d^2 s^2 \mathrm{e}^{T_d s} + 4T_d s\,\mathrm{e}^{T_d s} + 2\,\mathrm{e}^{T_d s} + 2k_1 r_1 = 0$$
$$T_d^2 s^2 + 6T_d s + 6 = 0$$

From the latest equation the quadruple dominant pole

$$s_4^* = -\frac{3-\sqrt{3}}{T_d}$$

can be obtained and successively the controller adjustable parameters

$$r_1^* = \frac{\sqrt{3}-1}{k_1}\mathrm{e}^{\sqrt{3}-3}$$
$$r_0^* = \frac{6(2\sqrt{3}-3)}{k_1 T_d}\mathrm{e}^{\sqrt{3}-3}$$
$$r_{-1}^* = \frac{6(7\sqrt{3}-12)}{k_1 T_d^2}\mathrm{e}^{\sqrt{3}-3}$$

can be obtained too.

For $r_1 = 0$ in the characteristic quasipolynomial (14) from the system of equations (15) for $i = 0, 1, 2$ the triple dominant pole

$$s_3^* = -\frac{2-\sqrt{2}}{T_d}$$

can be obtained and successively the controller adjustable parameters

$$r_0^* = \frac{2(\sqrt{2}-1)}{k_1 T_d}\mathrm{e}^{\sqrt{2}-2}$$
$$r_{-1}^* = \frac{2(5\sqrt{2}-7)}{k_1 T_d^2}\mathrm{e}^{\sqrt{2}-2}$$

After considering of the relations (7) for both controllers there can be obtained:

PI controller

$$k_P^* = r_0^* = \frac{2(\sqrt{2}-1)}{k_1 T_d}\mathrm{e}^{\sqrt{2}-2} \doteq 0{,}461\frac{1}{k_1 T_d}$$
$$T_I^* = \frac{r_0^*}{r_{-1}^*} = \frac{\sqrt{2}-1}{5\sqrt{2}-7}T_d \doteq 5{,}828 T_d$$

PID controller

$$k_P^* = r_0^* = \frac{6(2\sqrt{3}-3)}{k_1 T_d}\mathrm{e}^{\sqrt{3}-3} \doteq 0{,}784\frac{1}{k_1 T_d}$$
$$T_I^* = \frac{r_0^*}{r_{-1}^*} = \frac{2\sqrt{3}-3}{7\sqrt{3}-12}T_d \doteq 3{,}732 T_d$$

$$T_D^* = \frac{r_1^*}{r_0^*} = \frac{\sqrt{3}-1}{6(2\sqrt{3}-3)} T_d \doteq 0{,}263 T_d$$

Table 1: Adjustable parameters of PI and PID controllers for IPTD plants

METHOD	CONTROLLER	ADJUSTABLE PARAMETERS		
		k_P^*	T_I^*	T_D^*
ZIEGLER-NICHOLS (ZN)	PI	$0{,}714 \frac{1}{k_1 T_d}$	$3{,}333 T_d$	–
	PID	$0{,}924 \frac{1}{k_1 T_d}$	$2 T_d$	$0{,}5 T_d$
TYREUS-LUYBEN (TL)	PI	$0{,}491 \frac{1}{k_1 T_d}$	$8{,}8 T_d$	–
	PID	$0{,}714 \frac{1}{k_1 T_d}$	$8{,}8 T_d$	$0{,}635 T_d$
CHIDAMBARAM – SREE (ChS)	PI	$1{,}111 \frac{1}{k_1 T_d}$	$4{,}5 T_d$	–
	PID	$1{,}235 \frac{1}{k_1 T_d}$	$4{,}5 T_d$	$0{,}45 T_d$
MULTIPLE DOMINANT POLE (MDP)	PI	$0{,}461 \frac{1}{k_1 T_d}$	$5{,}828 T_d$	–
	PID	$0{,}784 \frac{1}{k_1 T_d}$	$3{,}732 T_d$	$0{,}263 T_d$

3 Example

The PI and PID analog controllers for the plant with the transfer function

$$G_P(s) = \frac{0{,}05}{s} \mathrm{e}^{-5s}$$

are necessary to tune by the above mentioned methods (time is in seconds).

Solution:

The adjustable parameter values can be obtained by substituting $k_1 = 0.05$ and $T_d = 5$ in the corresponding formulas in Tab. 1. The comparison of the tuning methods is shown in Fig. 2.

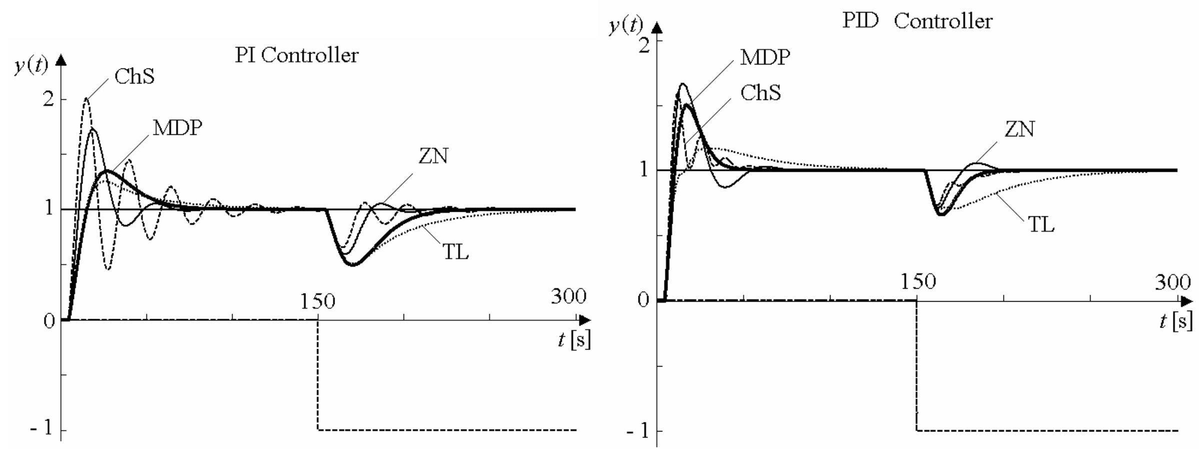

Fig. 2 – PI and PID controllers

The robustness of the tuning methods for ± 25 % changes of time delay T_d is shown in Fig. 3 to Fig. 6.

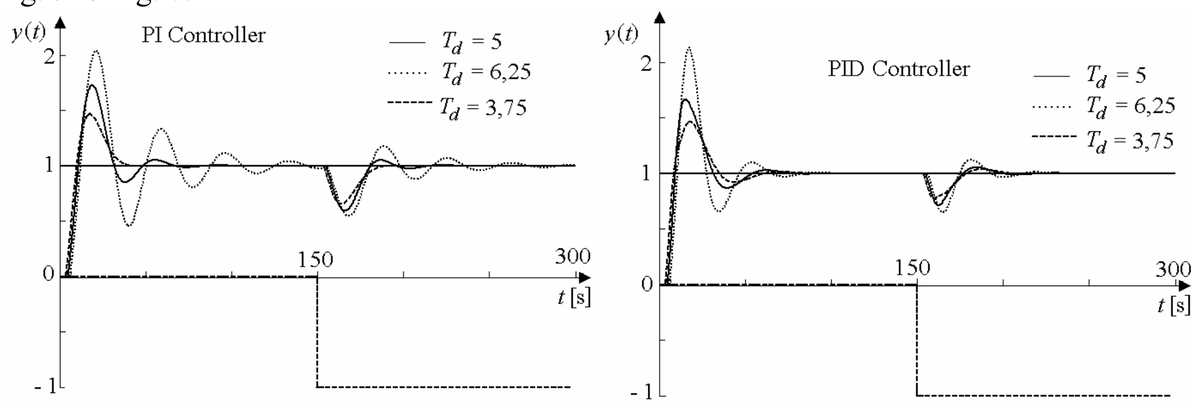

Fig. 3 – PI and PID controllers tuned by the ZN method

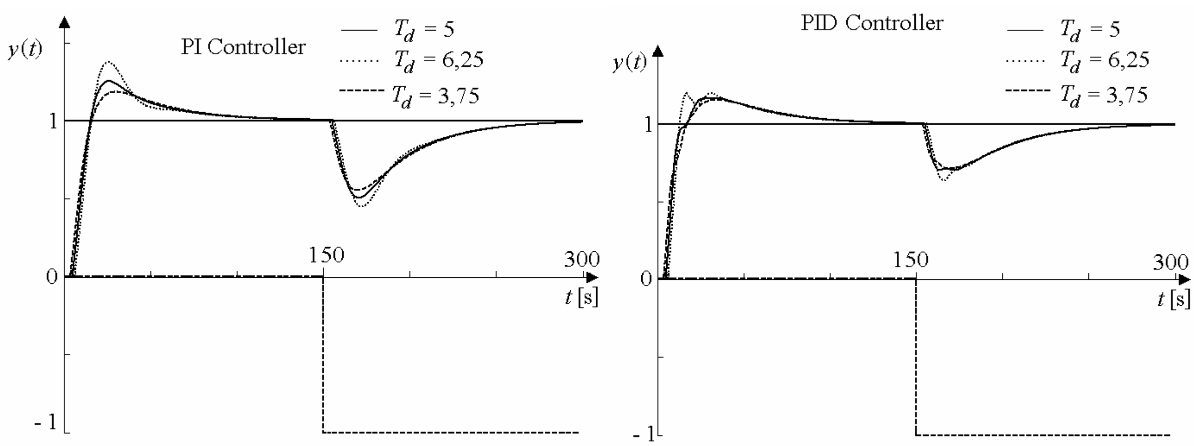

Fig. 4 – PI and PID controllers tuned by the TL method

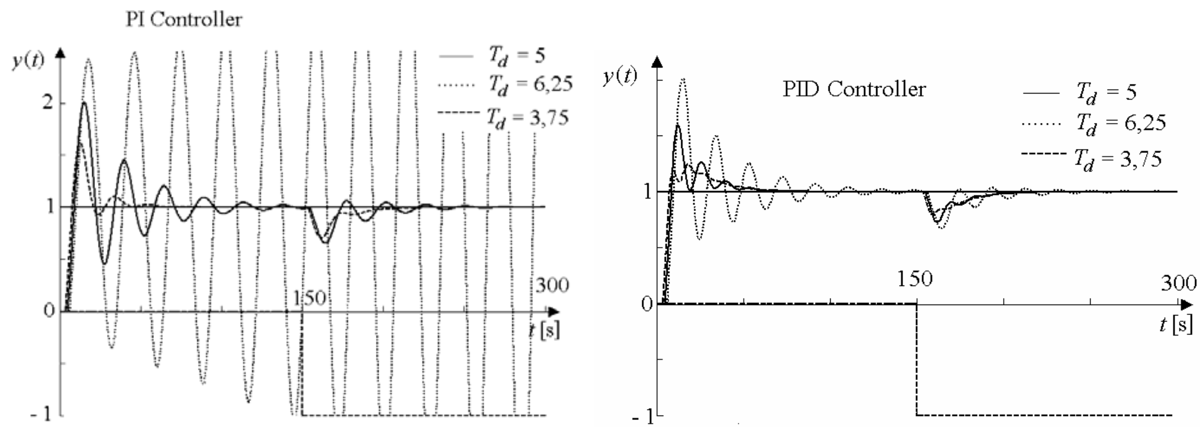

Fig. 5 – PI and PID controllers tuned by the ChS method

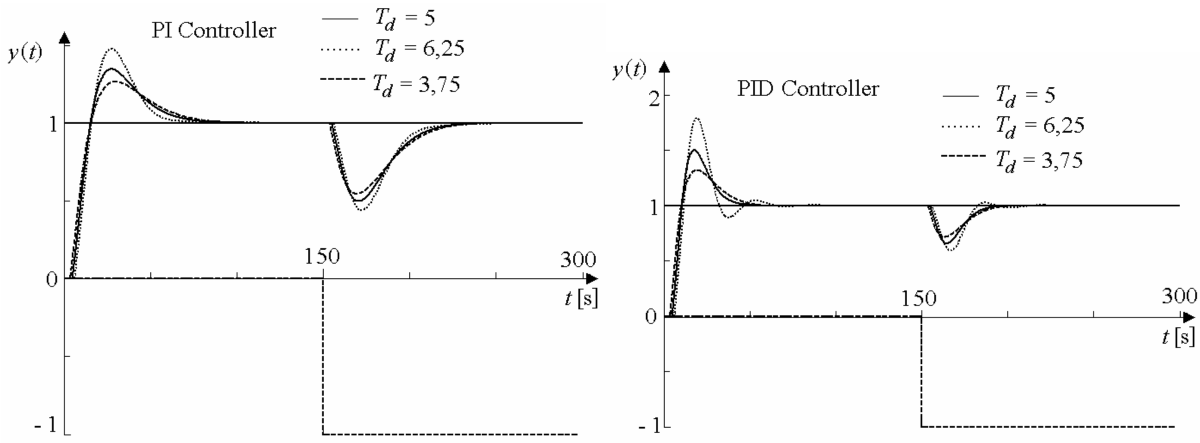

Fig. 6 – PI and PID controllers tuned by the MDP method

4 Conclusions

In this paper there are described and partially derived four simple methods for PI and PID analog controllers for the integral plus time delay plants. The multiple dominant pole method is new. It is shown that the Tyreus-Luyben method and the multiple dominant pole method give relatively robust both servo and load responses.

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