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Measurement of Positional Deviation of Numerically Controlled Axes

KUREKOVÁ, Eva, HALAJ, Martin, LOEBL, Tomáš & TVRDOŇOVÁ, Martina

doc. Ing., PhD.,

✉ Katedra automatizácie, informačnej a prístrojovej techniky, SjF STU, nám Slobody 17, Bratislava 1, 812 31, Slovenská republika, eva.kurekova@stuba.sk

doc. Ing., PhD., martin.halaj@stuba.sk

Ing., tomas.loebl@stuba.sk

PaedDr., martina.tvrdonova@stuba.sk

Abstract: *Nowadays many modern production machines require positioning in thousandths of millimeter. Therefore a big attention must be paid to prepare such a controlling software that drives individual axes of the production machine which minimises deviations between the desired position and actually reached position. The so called positional deviation (difference between the actual and target position) belongs to the important criteria that describe the performance of numerically controlled axes. The procedure for determination of such deviation is described in the international standard ISO 230-2:1997. This standard provides calculation of the positional deviation only in several discrete (measuring) points. Moreover it does not consider effects of the measuring instrument on the obtained results. Thus the new methodology must be adopted that enables estimation of the positional deviation in any point of the axis travel, together with the uncertainty of such estimate. Obtained results can be incorporated into a control system in the form of corrections enhancing positioning possibilities of individual axes.*

Keywords: *positional deviation, measured data, standard ISO 230-2:1997, numerically controlled axis*

1 Introduction

Testing of the positional deviation of the numerically controlled axis (either rotary or longitudinal) is ruled by the international standard ISO 230-2:1997 [2]. This standard provides guide for design of the test, testing conditions and also evaluation procedure for processing the measured data. In general, the testing procedure is based on repeated measurements of the actual position of the tested axis in several discrete points (target positions), located equally along the axis travel (see Figure 1).

The several parameters dealing with positional deviation can be measured and calculated according to such a scheme. The evaluation of measured data according to the standard gives just estimation of the device performance in several discrete points (measurement points P_i). But the course of individual parameters among the measurement points is just roughly estimated according to the standard, giving no warranty on correctness of the results in between points.

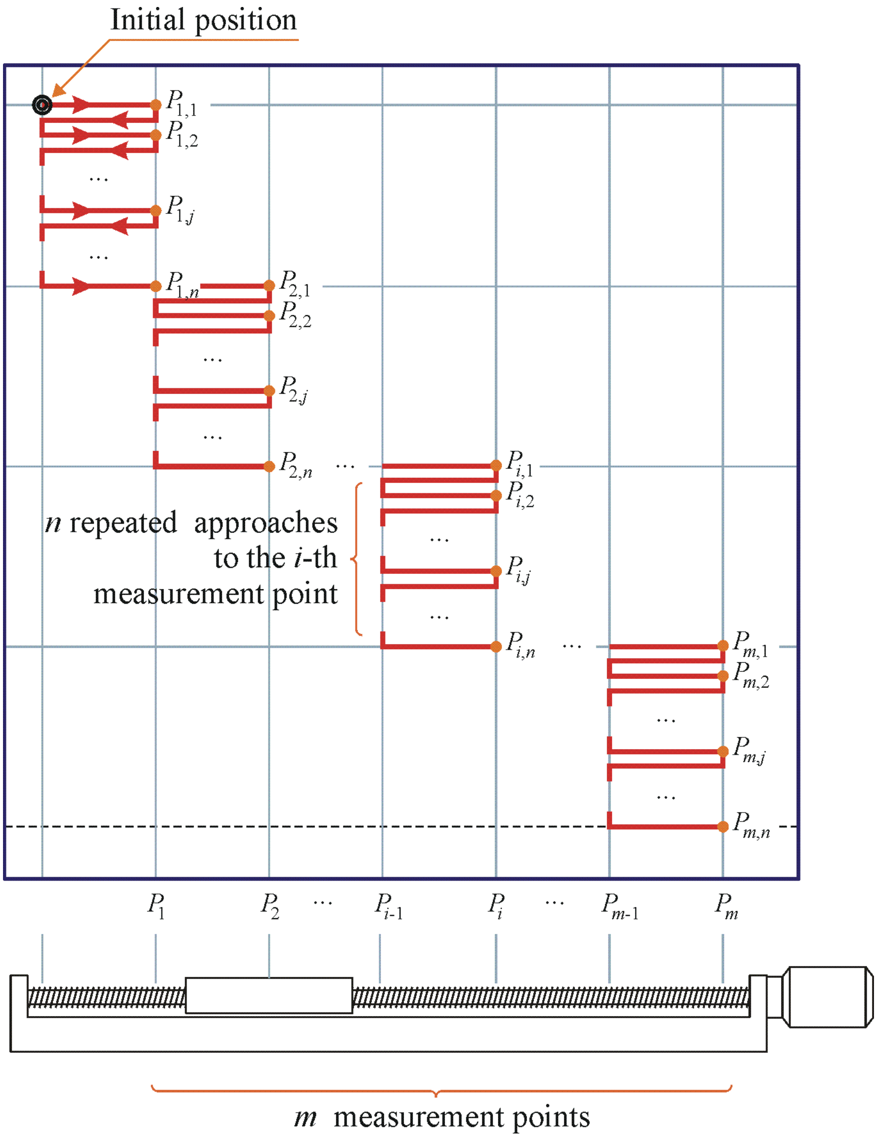

Figure 1 – Scheme of the measurement in individual points

2 Related terms

Terms connected with measurement of positioning numerically controlled axes, as introduced by the standard [2] (for sake of clarity only unidirectional approach is considered):

- *axis travel* – maximum travel, linear of rotary, over which the moving component can move under numerical control (par. 2.1),
- *measurement travel* – part of the axis travel which is used for data capture, selected so that the first and the last target positions may be approached bidirectionally (par. 2.2),
- *target position* - position to which the moving part is programmed to move (par. 2.3). Designation P_i ($i = 1$ to m),
- *actual position* – measured position reached by the moving part on the *j*-th approach to the *i*-th target position (par. 2.4). Designation P_{ij} ($i = 1$ to m, $j = 1$ to n),
- *deviation of position* (positional deviation) – actual position reached by the moving part minus the target position (par. 2.5). Designation x_{ij}, magnitude calculated as $x_{ij} = P_{ij} - P_i$,
- *mean unidirectional positional deviation* – arithmetic mean of the positional deviations obtained by a series of n unidirectional approaches to a position P_i (par. 2.10). Designation $\bar{x}_i \uparrow$, magnitude calculated as

$$\bar{x}_i \uparrow = \frac{1}{n}\sum_{j=1}^{n} x_{ij} \uparrow$$

- *estimator of the unidirectional standard uncertainty of positioning at a position* – estimator of the standard uncertainty of the positional deviations (par. 2.15). Designation $s_i \uparrow$, magnitude calculated as

$$s_i \uparrow = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} \left(x_{ij} \uparrow - \bar{x}_i \uparrow\right)^2}$$

- *unidirectional systematic positional deviation of an axis* – the difference between the algebraic maximum and minimum of the mean unidirectional positional deviations at any position P_i (par. 2.19). Designation $E \uparrow$,
- *unidirectional accuracy of positioning of an axis* (par. 2.22). Designation $A \uparrow$, magnitude calculated as $A \uparrow = \max\left[\bar{x}_i \uparrow + 2s_i \uparrow\right] - \min\left[\bar{x}_i \uparrow - 2s_i \uparrow\right]$.

3 Evaluation of measured data according to the standard

The above mentioned standard introduces evaluation of the measured data that is aimed namely at determining the maximum positional deviation over the whole axis (measurement) travel. The evaluation of results covers calculation of the parameters related to the positional deviation (see part 2) in each of the measurement points P_i, covered also by the deviation boundaries $\bar{x}_i \uparrow$ +3 $s_i \uparrow$; $\bar{x}_i \uparrow$ -2 $s_i \uparrow$ (respectively $\bar{x}_i \downarrow$ +2 $s_i \downarrow$; $\bar{x}_i \downarrow$ -3 $s_i \downarrow$ in reversal direction), see Figure 2.

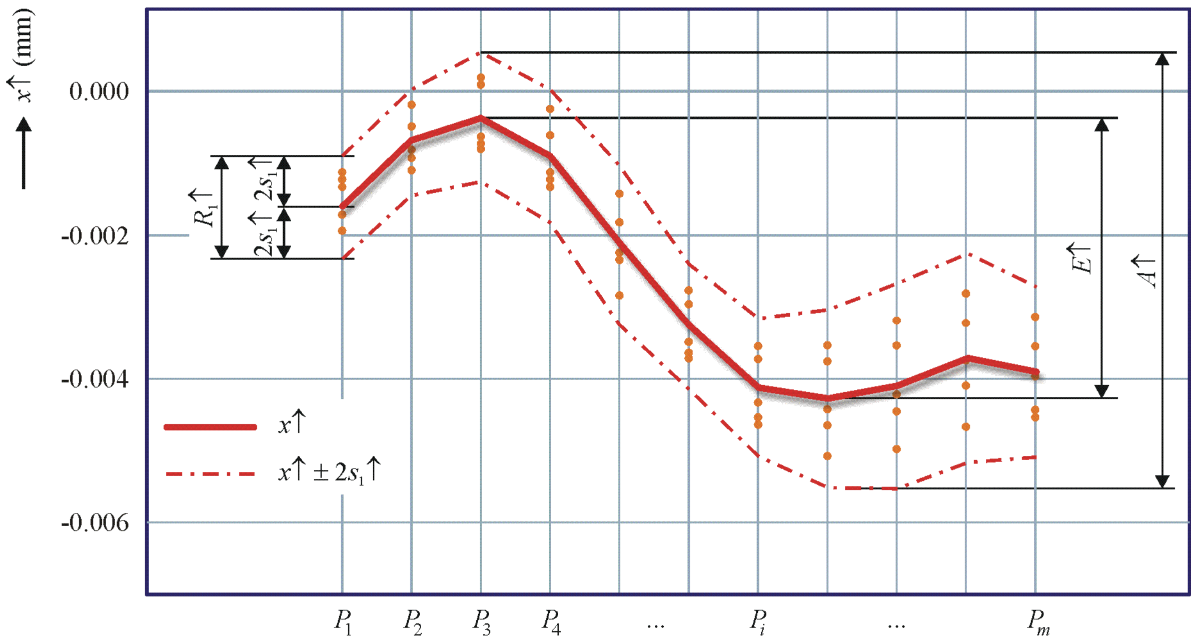

Figure 2 – Example of the graphical presentation of evaluated positional deviation according to the standard [2] (results plotted for unidirectional positioning)

Note that the deviation boundaries are calculated only from measured data, considering their variation as caused only by a tested machine and not influenced by the measurement instrument itself. This is valid only when the accuracy of measurement instrument is much better than expected positional deviations. But for modern numerically controlled axes, operating with resolutions of 0.001 mm, this is not always the case and the used measuring instrument itself can significantly affect the measurement results.

As you can observe from Figure 2, the standard assumes linear course of the positional deviation among the measurement (testing point), as well as linear course of the deviation boundaries.

4 Evaluation of the positional deviation in any point of the axis travel

When considering the above presented measurement scheme, following positional deviations are measured in individual measurement points:

$$\begin{aligned}
&P_1:\ \Delta_{11}, \Delta_{12}, ..., \Delta_{1j}, ..., \Delta_{1n}\\
&P_2:\ \Delta_{21}, \Delta_{22}, ..., \Delta_{2j}, ..., \Delta_{2n}\\
&\vdots\\
&P_m:\ \Delta_{m1}, \Delta_{m2}, ..., \Delta_{mj}, ..., \Delta_{mn}
\end{aligned} \tag{1}$$

where i – to m is the running number of the measurement point (see Figure 1),

Δ_{ij} – are positional deviations* (see part 2),

j – 1 to n is the running number of the measurement of positional deviation in a given measurement point. It is assumed that the same number n of measurements of the positional deviation is performed in each measurement point.

(*Remark: the positional deviation that is designated in the standard [2] as x_{ij}, is for sake of better clarity and understandability designated by different symbol Δ_{ij}).

If we want to obtain the estimates of the positional deviations also in other points than the measurement ones, we must approximate course of estimates. The least squares method is suitable for such approximation. The curve in a form of polynomial of the third order will be placed over the points $(P_1, \overline{\Delta_1})$, $(P_2, \overline{\Delta_2})$, ..., $(P_i, \overline{\Delta_i})$, ..., $(P_m, \overline{\Delta_m})$:

$$\Delta = a + b \cdot P + c \cdot P^2 + d \cdot P^3 \tag{2}$$

where Δ – is the positional deviation of the target position and actual position in any point P and $P \in \langle P_1; P_m \rangle$,

a, b, c, d – are unknown parameters of the polynomial.

Besides that we want to determine also expanded uncertainty U of estimate of the positional deviation Δ in any point P. To be able to do this, we must determine the estimates of polynomial parameters a, b, c and d, their uncertainties and covariances among them. Thus we leave the evaluation according to the Figure 2 and we get evaluation providing results according to the Figure 3. To do so, we need to introduce a completely new approach to evaluation of the measured data.

The positional deviation for each measurement j = 1 to n in each particular point P_i, with i = 1 to m, can be calculated as the difference between the target position and the measured actual position (see part 2 above) [2]:

$$\Delta_{ij} = P'_{ij} - P_i \tag{3}$$

where P_i – is the target (programmed) position,

P'_{ij} – is the actual (measured) position.

The actual (measured) position P'_{ij} comprises two components:

$$P'_{ij} = P_{ij} + \delta_{mer.} \tag{4}$$

where P_{ij} – is the position indicated by the measuring instrument,

$\delta_{mer.}$ – is the measurement error in the particular point, in our case estimated as the maximum permissible error of the measuring instrument [8]. This means that the error remains constant in any point.

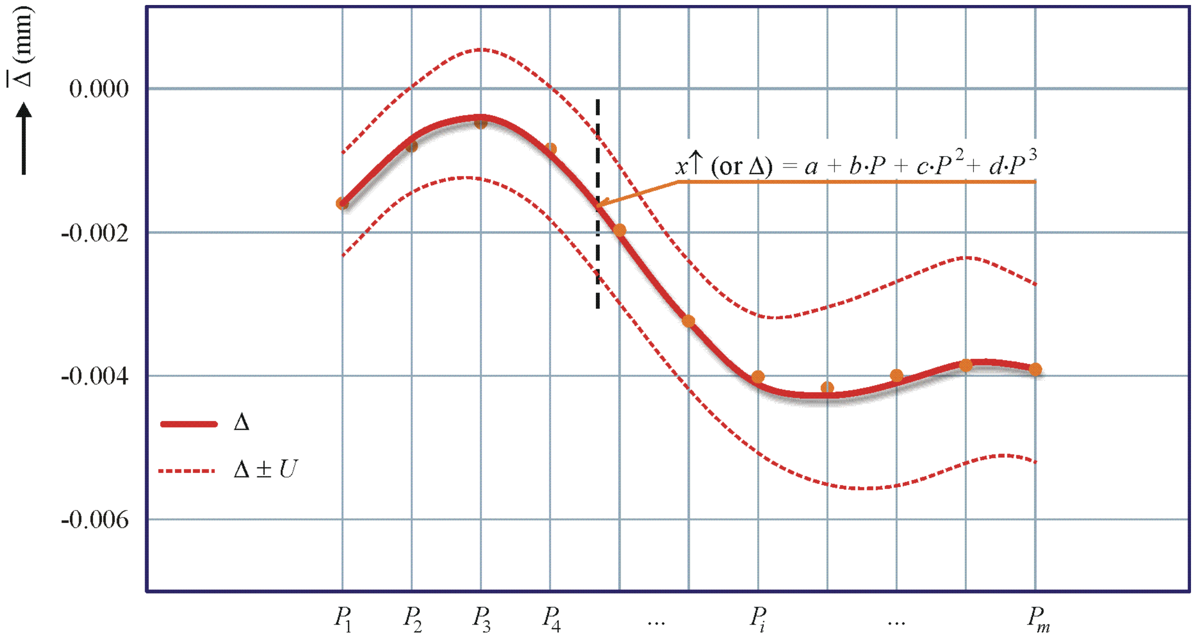

Figure 3 – Estimation of the positional deviation in any point of the axis travel

According to the previous calculations of estimates of the positional deviation (as the arithmetic means), the set of equations for j = 1 to n positional deviations in m points (target positions) can be expressed as [5]:

$$
\begin{aligned}
\overline{\Delta}_1 &= \overline{P}_1 - P_1 + \delta_{\text{mer.}} \\
\overline{\Delta}_2 &= \overline{P}_2 - P_2 + \delta_{\text{mer}} \\
&\vdots \\
\overline{\Delta}_m &= \overline{P}_m - P_m + \delta_{\text{mer.}}
\end{aligned}
\tag{5}
$$

where $\overline{P}_i$ – is the estimate (obtained as an arithmetic mean) of the actual (measured) positions in any given point P_i.

The set of equations describing the positional deviations (5) can be written in a matrix form:

$$\boldsymbol{x} = \overline{\boldsymbol{P}} - \boldsymbol{i}{\cdot}\boldsymbol{P} + \boldsymbol{i}{\cdot}\delta_{\text{mer}} \tag{6}$$

where $\boldsymbol{x}$ – is the vector of the estimates of individual positional deviations (dimension m),

$\overline{\boldsymbol{P}}$ – is the vector of the estimates of measured actual positions (dimension m),

$\boldsymbol{P}$ – is the vector of target positions (dimension m),

$\boldsymbol{i}$ – is the unit vector (dimension m) and

$$\boldsymbol{x} = \left(\overline{\Delta}_1, \overline{\Delta}_2, \cdots, \overline{\Delta}_m\right)^{\mathrm{T}}$$

$$\overline{\boldsymbol{P}} = (\overline{P}_1, \overline{P}_2, ..., \overline{P}_m)^{\mathrm{T}}$$

$$\boldsymbol{P} = (P_1, P_2, ..., P_m)^{\mathrm{T}}$$

When taking the matrix notation into account, the covariance matrix $\boldsymbol{U}(\boldsymbol{x})$ can be written in a form (assuming $\boldsymbol{P}$ as non-random vector, $\overline{\boldsymbol{P}}$ and δ are independent):

$$\boldsymbol{U}(\mathbf{x}) = \boldsymbol{U}(\overline{\boldsymbol{P}}) - \boldsymbol{U}(\boldsymbol{P}) + u^2(\delta)\boldsymbol{i}\,\boldsymbol{i}^T \tag{7}$$

The uncertainty of the target position is zero in our case (no influence on value of the target position) so that the covariance matrix $\boldsymbol{U}(\boldsymbol{x})$ gets the form [4]:

$$\boldsymbol{U}(\boldsymbol{x}) = \boldsymbol{U}(\overline{\boldsymbol{P}}) + u^2(\delta)\boldsymbol{i}\,\boldsymbol{i}^T \tag{8}$$

After specifying the individual terms (because P_i a P_j are independent):

$$\boldsymbol{U}(\boldsymbol{x}) = \begin{pmatrix} u^2(P_1) & 0 & \cdots & 0 \\ 0 & u^2(P_2) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & u^2(P_m) \end{pmatrix} + u^2(\delta)\, i\, i^{\mathrm{T}} =$$

$$= \begin{pmatrix} u^2(P_1)+u^2(\delta) & u^2(\delta) & \cdots & u^2(\delta) \\ u^2(\delta) & u^2(P_2)+u^2(\delta) & \cdots & u^2(\delta) \\ \cdots & \cdots & \cdots & \cdots \\ u^2(\delta) & u^2(\delta) & \cdots & u^2(P_m)+u^2(\delta) \end{pmatrix} \tag{9}$$

The uncertainties $u(P_i)$, where $i = 1, 2, ..., m$, in the matrix (8) are evaluated by the type A method form measured data:

$$u(P_i) = \sqrt{\frac{1}{n(n-1)}\sum_{j=1}^{n}(P_{i,j} - \overline{P}_i)^2} \tag{10}$$

This procedure yields to the estimates of unknown parameters a, b, c, d, uncertainties of those estimates and covariances among them.

5 The experiment

The experimental measurement of positional deviation was performed at the laboratories of Department of Production Technology, Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava [3]. The prototype of the universal plasma cutting head [1] was employed for measurement of individual positional deviations (see Figure 4., - Figure is courtesy of the Department of Production Technology, Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava). As the plasma cutting head contains totally 6 numerically controlled axis, the experiment was restricted to one axis and the others were in standstill. The schematic representation of the measured axis together with the precise linear encoder (used as a reference measuring instrument) is shown in Figure 5.

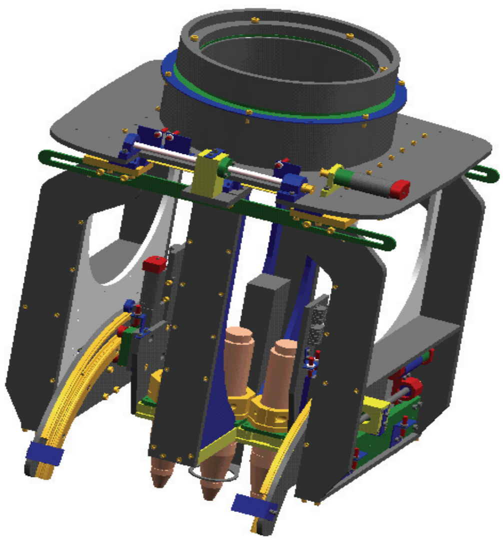

Figure 4 – Universal plasma cutting head

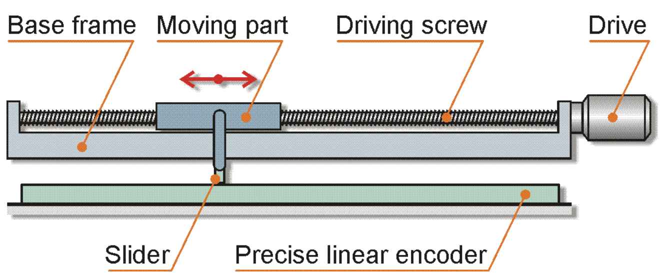

Figure 5 – Schematic representation of the measured axis

The preliminary results, evaluated according to the standard so far, are shown in Figures 6 to 8. One can observe that the course of approach does not significantly affect the obtained results. Due to a lack of time the evaluation according to a new methodology has not been performed yet but it is expected in a short time. The presented methodology is adapted from the methods used for calibration of measuring instruments; therefore its correctness has already been proved [6], [7].

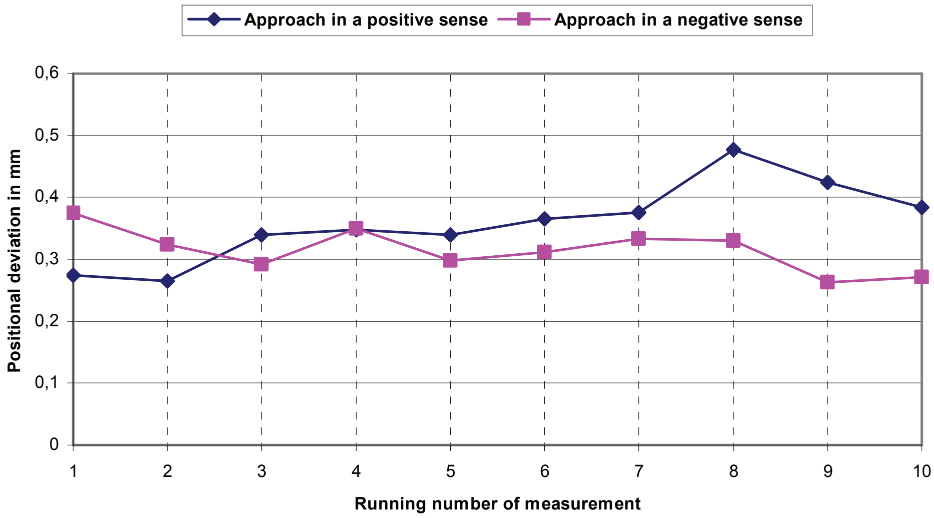

Figure 6 – Positional deviation measured when approaching to individual target points from both senses of movement

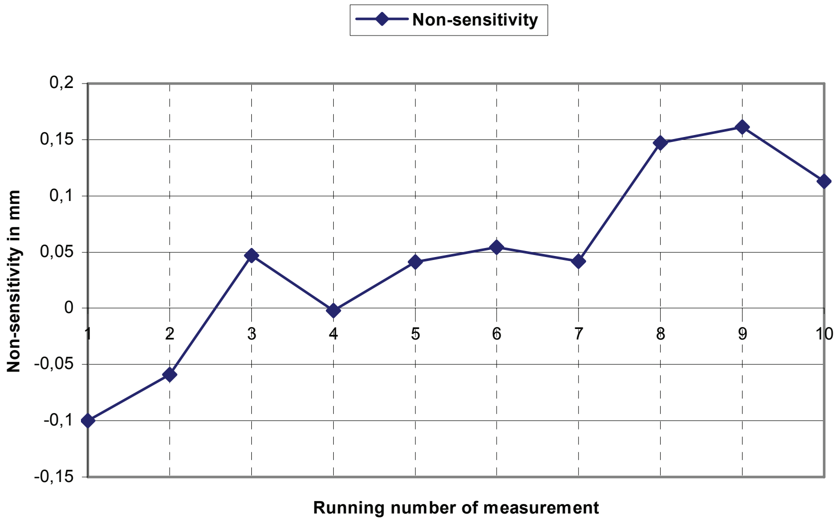

Figure 7 – Non-sensitivity of the CNC axis

6 Conclusions

The presented methodology gives the opportunity to estimate the positional deviation in any point of the axis travel, no matter whether rotational or longitudinal. Moreover it provides the estimate of the positional deviation with the respective uncertainty of such estimate. This gives the designer or programmer the possibility to build appropriate corrections into the control program or the adequate design corrections can be performed in the design of the machine.

The presented evaluation of measured data according to the standard shows similar behavior of the controlled axes when approaching to the desired position from both sides (positive and negative).

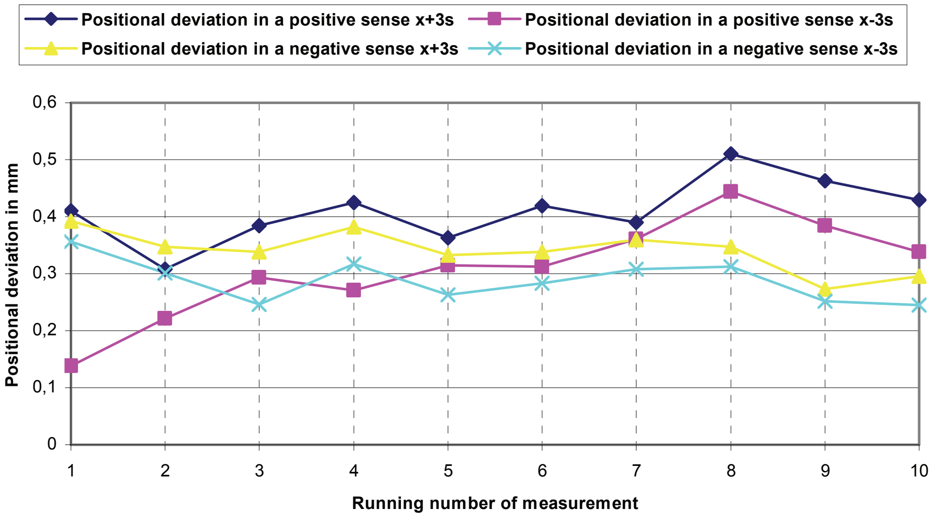

Figure 8 – Boundaries of the positional deviation in both senses of motion

Acknowledgements:
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