XXXII. Seminar ASR '2007 "Instruments and Control", Farana, Smutný, Kočí & Babiuch (eds)
© 2007, VŠB-TUO, Ostrava, ISBN 978-80-248-1272-4

The Mathematical Model of Contraction Characteristic $k = (F, p)$ of the Pneumatic Artificial Muscle

BORŽÍKOVÁ, Jana, BALARA, Milan & PITEĽ, Ján

Mgr., PhD., Doc., PhD., Ing., PhD. FVT TU Košice, Department of Mathematics, Informatics and Cybernetics, Bayerova 1, Prešov, 080 01 borzikova.jana@fvt.sk, balara.milan@fvt.sk, jan.pitel@tuke.sk http://www.tuke.sk/fvtpo/

Abstract: *The article concerns of the mathematical model of static properties of the pneumatic artificial muscles, which are shown as static characteristics in this article. The mathematical model was derived from measuring of pneumatic artificial muscles Shadow Air Muscle (SAM 30 x 290). Such mathematical interpretation of static characteristic is pass current and enables their implementation in complete mathematical model of the pneumatic artificial muscle system.*

Keywords: *pneumatic artificial muscle, static characteristics, contraction*

1 Pneumatic artificial muscles

Pneumatic artificial muscles are progressive actuators of pneumatic and electropneumatic servosystems. Such servosystems are irreplaceable in certain applications (for example, explosive environment). This mathematical model of pneumatic muscles was created on the basis of measurments of pneumatic artificial muscles Shadow Air Muscle (SAM 30 x 290). The mathematical interpretation of static characteristic generally allows to include them into complete mathematical model of pneumatic artificial muscle system.

They possess several advantages over other types of actuators but most of all, it is their power-to-weight ratio. Acquisition cost is relatively low. Muscules are suitable for application to stationary and mobile devices, where both low weight and low cost at either standard or exceptional technical device parameters are required.

The most made and common type of artificial muscle in use is the McKibben artificial muscle. This type is similar in properties to biological muscle. The principle is very simple: The muscle consists of a rubber tube on the surface of which longitudinal nylon filaments forming netting are placed. Suitable material for making of the pneumatic artificial muscles is latex and silicone gum, fibres are from nylon. When inflated with compressed air, the tube extends causing a simultaneous extension and axial contraction of the length of netted nylon filaments found on its surface. Thus the contraction of the whole artificial muscle occurs. The magnitude of this contraction depends on the air pressure and the time of air flowing into the muscle or deflating artificial muscle. When a load of muscle is increased, its pull force rises while the lift of a muscle decreases. (Tondu & Lopez, 2000), (Chou & Hannaford, 1996).

The advantages of this type of muscles become readily evident when we compare weight of pneumatic muscles with weight of pneumatic cylinder. The total weight is only 10 % of pneumatic cylinder weight with the same diameter. Initial force during contraction is 10 times bigger. Pneumatic artificial muscle works without stick-slip effect and therefore small and slow motion are possible. The construction of these muscles allows good dynamic performance and position control. This construction is used by several world firm, for example the firm Festo declares top parameter of artificial muscles: force up to 6000 N, acceleration 100 $m.s^{-2}$ and minimal motion speed up to 10^{-3} mm/s. (Hošovský, 2006)

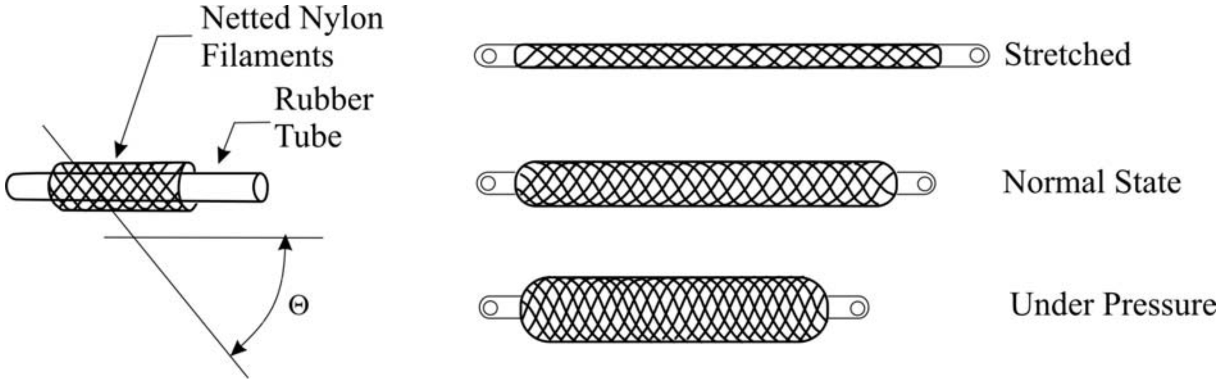

Figure 1 – The pneumatic artificial muscle as McKibben type developed by doctor J. L. McKibben for pneumatic prosthesis

2 Shadow Air Muscles

Pneumatic artificial muscles of the SAM type are supplied by a British company Shadow Robot Company. A group of 4 muscles with different diameters (12, 20, 30 and 50 mm) and lengths (150, 210, 290 and 400 mm) are available. They achieve contraction of 25% under loads of 3, 12, 30 and 75 kg. SAM artificial muscles are modification of McKibben and the basic difference between artificial muscles of the SAM type and the McKibben type is that SAMs require a certain load at initial position. Contraction depends on pressure of air and speed of changes.

Table 1. An assortment of Shadow Air Muscle

Pictures	Diameter	Length	Weight	Force (3.5 bar)	Max. force
	6 mm	150 mm	10 g	3 kg	7 kg
	20 mm	210 mm	40 g	12 kg	20 kg
	30 mm	290 mm	80 g	35 kg	70 kg

3 The static characteristic of pneumatic artificial muscles

Generally, the pneumatic artificial muscles properties and their mathematical models are represented by relatively complicated non-linear system with several non-linearities and time delay. (Chou – Hannaford, 1996) The modelling and simulation of such non-linear systems is realized by simulation programs to obtain static characteristic, transient response and other types of characteristic.

The behaviour of the contraction static characteristics is system of curves, which presents dependence between filling pressure *p* and contraction *k* at varying tensile force *F*. Each of the characteristics appears as non-linearity consisting of continuous non-linear curve of type of saturation with dead zone (Fig. 2)

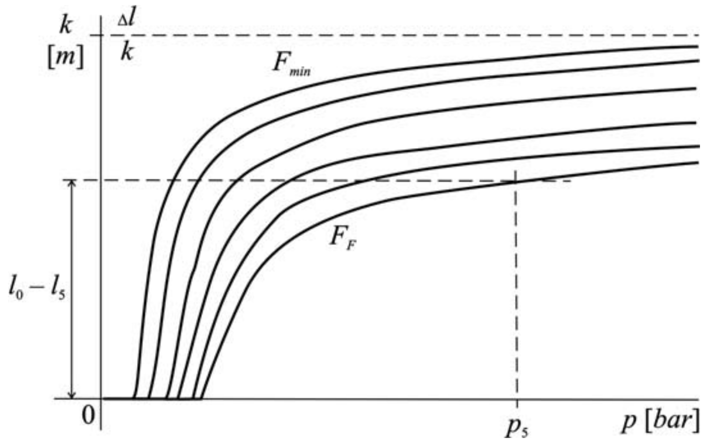

Figure 2 – Transient responses $k = f(p)$ at varying tensile force F

4 Mathematical model of contraction static characteristic of the pneumatic artificial muscle SAM 30 x 290

Based on experiments the following pressure-contraction characteristics of the SAM 30x290 artificial muscle were measured at varying tensile force F. To simplify conditions of solution, filling and emission are substituted by average of contraction values. (Fig. 3.)

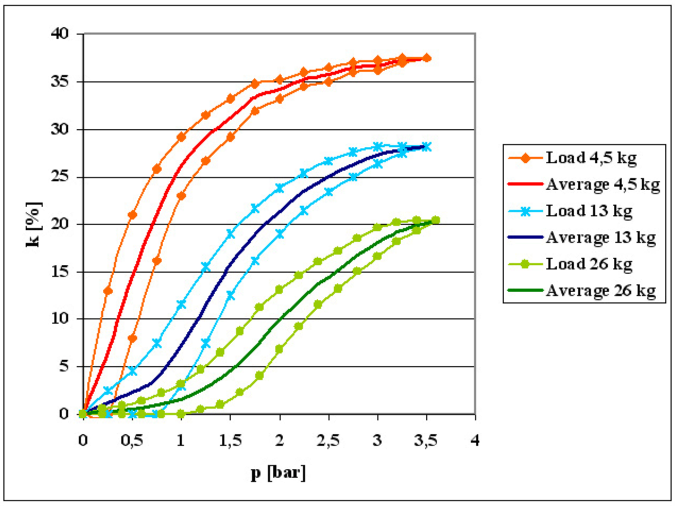

Figure 3 – Measured values $k_i = f(p_i, F_i)$

Firstly, we calculated functions on plane. Acquired approximations are

$$k = 40 - 40.e^{0{,}1273p^2 - 1{,}2602p + 0{,}0856}$$

with correlation index IK = 0,998 for tensile force F = 4,5 kg. Secondly

$$k = 40 - 40.e^{0{,}0742p^3 - 0{,}3787p^2 + 0{,}0872p - 0{,}0082}$$

with correlation index IK = 0,999 for tensile force F = 13 kg. And finally

$$k = 40 - 40.e^{0{,}0127p^4 - 0{,}0607p^3 - 0{,}0034p^2 + 0{,}0147p + 0{,}007}$$

with correlation index I = 0,999 for tensile force F = 26 kg. (Fig. 4)

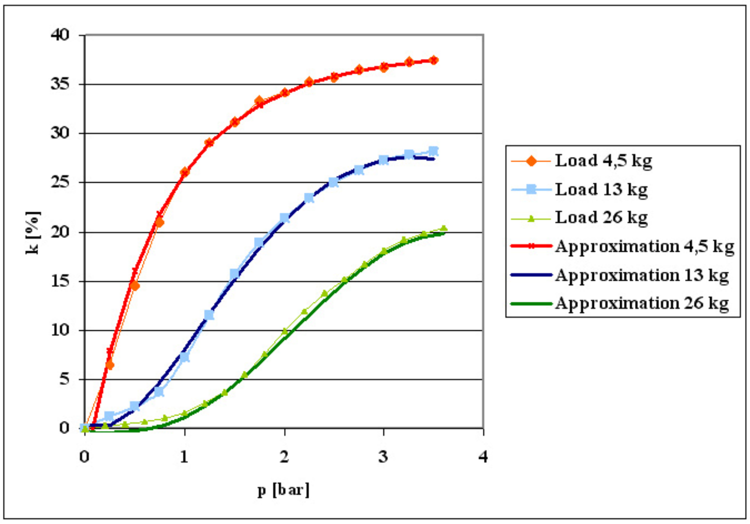

Figure 4 The behaviour of the function $k = f(p)$ obtained by approximation of measured values (various weights F).

On the basis of the solution of task in a plane at varying tensile force F (45 N, 130 N, 260 N), suitable approximation of measured values [p_i, F_i, k_i] appears function

$$k = f(p, F, \mathbf{A}) = 40 - 40.e^{P(p,F)} \tag{1}$$

where $P(p, F)$ is polynomial $\sum_{i=0}^{4}\sum_{j=0}^{2} a_{ij} p^i F^j$ and $\mathbf{A} = (a_{i,j})$ is matrix of desiderative coefficients.

We used the least square method (algorithm of linearization) to find unknown parameters. They are determined in such way that the surface approximates values as precisely as possible. To meet this requirement, a minimum of function is taken for criterion

$$S(A) = \sum_{i=1}^{n} \left[k_i - f(p_i, F_i, \mathbf{A})\right]^2 \tag{2}$$

Quality of substitution is defined by the correlation index (Dávid et al, 2000):

$$IK = \sqrt{1 - \frac{\sum_{i=1}^{n} (k_i - y_i)^2}{\sum_{i=1}^{n} (\bar{k} - y_i)^2}} \tag{3}$$

where k_i denotes evaluated values and $\bar{k}$ is the average of measured values.

MS Excel was used for solution by chosen method and measured values (Fig. 3) were approximated by function

$$k = 40 - 40.e^{v}, \tag{4}$$

where v je $\sum_{i=0}^{4}\sum_{j=0}^{2} a_{ij} p^i F^j$ and $a_{i,j}$ is matrix of results:

$$a_{i,j} = \begin{pmatrix} -1,888 & 0,0283 & -7,55.10^{-5} \\ 73,4973 & -0,917 & 0,0024 \\ -42,073 & 0,7612 & -0,0023 \\ 9,9754 & -0,2266 & 7,3257.10^{-4} \\ -0,8231 & 0,0228 & -7,902.10^{-5} \end{pmatrix}$$

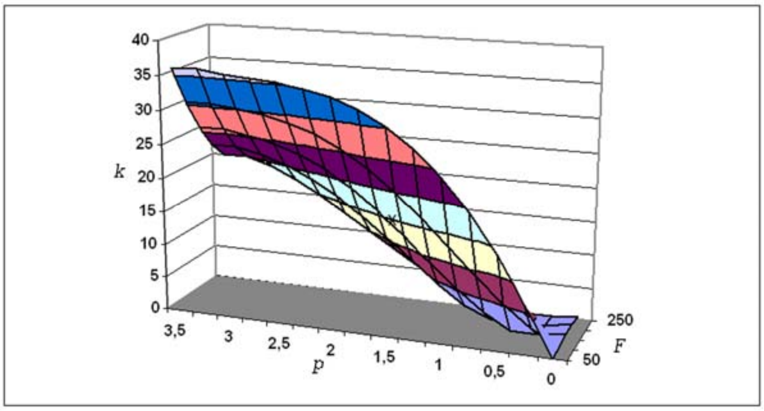

Figure 5 The behaviour of the function $k = f(p, F)$ obtained by approximation of measured values [p_i, F_i, k_i]

5 Conclusion

The obtained results of the static characteristic function of the pneumatic artificial muscle as SAM 30 x 290 type can be generalized. It follows from results above we can observed, that pneumatic artificial muscles function, which describes contraction *k* on pressure *p* and tensile force *F* is:

$$k = k_{\max}.\left(1 - e^{v}\right),$$

where k_{max} is maximum value of contraction, *v* is $\sum_{i=0}^{4}\sum_{j=0}^{2} a_{ij} p^{i} F^{j}$ and $a_{i,j}$ is matrix of results. The coefficients of matrix depend on the material properties of the pneumatic artificial muscle and its geometric dimension. Finally, we can say that it is necessary perform measurements of concrete chosen pneumatic artificial muscle to find coefficient matrix and static characteristic of chosen muscles.

This contribution work was performed to financial support of project VEGA no. 1/2212/05 „The research and development of mechatronic components and systems of bioservosystems based on the pneumatic artificial muscles“ and no. 1/4077/07 „Research of algorithms for optimization of control of non-linear dynamic systems with time delay by artificial intelligence techniques“.

6 References

BALARA, M. & PETÍK, A. 2001. Experimentálne meranie statickej charakteristiky umelého svalu MB0124. In: *New Trends in the Operation of Production Technology*. 4 – th international scientific conference. 21. – 22. 11. 2001, Prešov, Slovakia: FVT TUKE, 2001, pp. 97 - 100. ISBN 80 – 7099 – 723 – 0

DÁVID, A. et al. 2000. *Vybrané numerické metódy a ich programovanie v Exceli.* Bratislava: Ekonóm, 2000. 180 s. ISBN 80-225-1280-X

HOŠOVSKÝ, A. 2006. *Polohové systémy na báze pneumatických umelých svalov a možnosti ich zdokonaľovania*, Písomná práca k dizertačnej skúške, Prešov: FVT TU Košiciach so sídlom v Prešove, 2006, pp. 5-20.

CHOU, C. P. & HANNAFORD, B. 1996. Measurement and modeling of artificial muscles. In: *IEEE Transactions on Robotics and Automation*, 1996,Vol. 12, pp. 90-102

KALIŠ, J. 2000. *Excel, učebnice programování*. Praha: GComp, 2000. 232 s. ISBN 80-85649-33-0

TONDU, B. & LOPEZ, P. 2000. Modelling and Control of McKibben Artificial Muscle Robot Actuators, In: *IEEE Control systems Magazine*, April 2000, pp. 15- 38